Stability Analysis of Analog Frequency Dividers in the Quasi-Periodic Regime

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***Abstract*—A simulation technique allowing the determination of all the steady state regimes of synchronized devices, including the quasi-periodic operation, is presented here. Combined with a continuation method, it provides the circuit quasi-periodic solution paths as a function of any parameter of interest. The different bifurcation loci on a two-parameters space, obtained here both from periodic and quasi-periodic simulations, allow an easy prediction and deep understanding of the circuit behavior. By means of the proposed method, the possible working regimes of a M.M.I.C analog frequency divider have been determined and experimentally confirmed.**

I. Introduction

In the first papers on injection-locked oscillators appearing in microwaves literature [1], [2], [3], the synchronized behavior of these devices was analyzed by means of the description function, the validity of the results limited by the initial assumptions of quasi-static behavior and single nonlinearity. Later, the harmonic balance technique has allowed a general analysis [4], [5] of the periodic responses of injected oscillators and frequency dividers, applicable regardless of the number of harmonic components and nonlinearities considered in the circuit. Outside the locking ranges, the quasi-periodic response of regenerative circuits was analytically predicted by Adler [6], under the assumption of quasi-static behavior. Recent works [7], [8] have generalized these results by means of the spectral balance technique, presenting the full simulation of an injection-locked oscillator, that includes the global stability analysis of the solution paths in the sub-synchronized region [7].

Here, this method is extended to provide the bifurcation locus that corresponds to the inverse transformation from quasi-periodic to periodic regime. This completes the global characterization of analog frequency dividers and phase-locked oscillators, allowing the immediate determination of the circuit acquisition band and frequency locking range in terms both of input power and frequency.

The method has been applied to a MMIC frequency divider, yielding an excellent agreement with experimental results. Bifurcation loci on a two-parameters plane from an autonomous quasi-periodic regime (self oscillator mixer) are presented here, for the first time, with experimental verification.

Manuscript received January 6, 1994.
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IEEE Log Number 9401062.

II. Analysis Method

Let us consider a free-running oscillator at the frequency ω_0, to which an external synchronizing source of frequency ω_{in} is now connected, and let us assume that under a certain threshold value of the input power *Pin* the circuit is unlocked. In these conditions, the circuit behaves in quasi-periodic regime with two independent fundamentals: ω_{in} and ω_p, ω_p being the autonomous frequency modified under the influence of the input signal.

The steady state spectrum of the unlocked regime will be thus composed of all the intermodulation products spanned by the two fundamentals. For numerical computation, this spectrum must be restricted to a finite number of frequency lines, which can be done by limiting the frequency bandwidth taken into account around DC and the first harmonics of the input signal. Only the intermodulation products $m\omega_{in} + n\omega_p$, with m, n integers belonging to one of the selected bands will thus be considered for the analysis.

In order to avoid the risk of convergence towards the trivial solution, a measuring probe working [5], [8] at the unknown frequency ω_p will be introduced into the circuit. The spectral balance equations, associated with the probe nonperturbation condition of the steady state, yield the nonlinear system:

$$\bar{H}_s(\bar{X}, E_p, \phi_p, \omega_p) = 0 \tag{1}$$

where X is the vector containing the spectral components of the nonlinear sources and E_p, ϕ_p and ω_p are, respectively, the amplitude, relative phase, and frequency of the measuring probe. There is one more unknown than number of equations, so one of the three magnitudes determining the probe value must be initially fixed, depending on the operation regime. In the quasi-periodic regime, due to the phase independence between the input generator and the autonomous signal, the probe phase can arbitrarily be made equal to zero. The system (1) is solved through the Newton-Raphson algorithm, being the spectre $m\omega_{in} + n\omega_p$ recalculated and realigned for every variation of the natural frequency.

In order to determine the global circuit behavior, a parametrization is introduced into the system, determining its bifurcation points by solving the general bifurcation system [5]:

$$\begin{aligned} \bar{H}_s(\bar{X}, \omega_p, P_{in}, \omega_{in}) &= 0 \\ \Delta(\bar{X}, \omega_p, \Omega) &= 0 \end{aligned} \tag{2}$$

where P_{in} and ω_{in} are the system parameters, Ω the perturbation frequency, and Δ the determinant of the characteristic matrix.

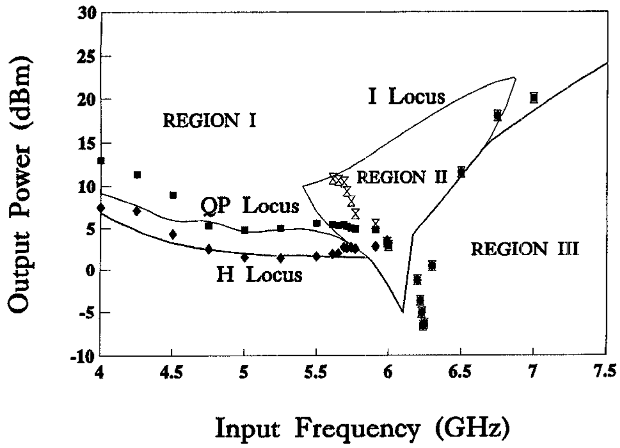


Fig. 1. Bifurcation loci. Rhombic points: measurements of periodic to quasi-periodic transformation. Square points: measurements of quasi-periodic to periodic transformation. Butterflies: frequency division.

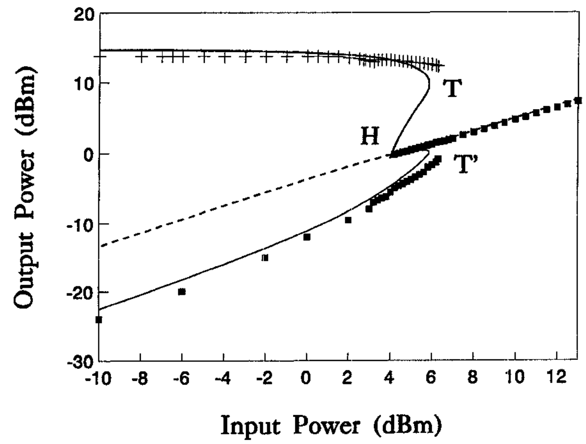


Fig. 2. Bifurcation diagram for input frequency 4.75 GHz. Upper curve: output power at the injection frequency. Lower curve: output power at the autonomous frequency. Straight line: output power at the injection frequency from periodic simulation. Square points and crosses are measurements.

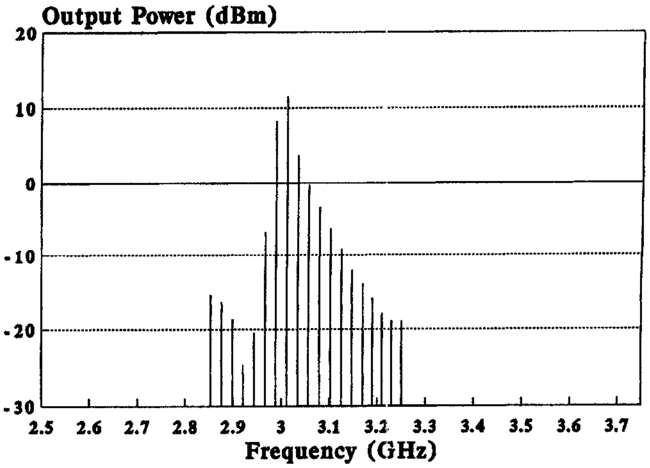
The set of solutions of (2) yields the bifurcation loci in the $[\omega_{in}, P_{in}]$ plane, which are automatically obtained through a continuation method with parameters exchange. The different loci will be classified according to the values of ω_p and Ω, as indicated in the following example.

III. Monolithic Frequency Divider

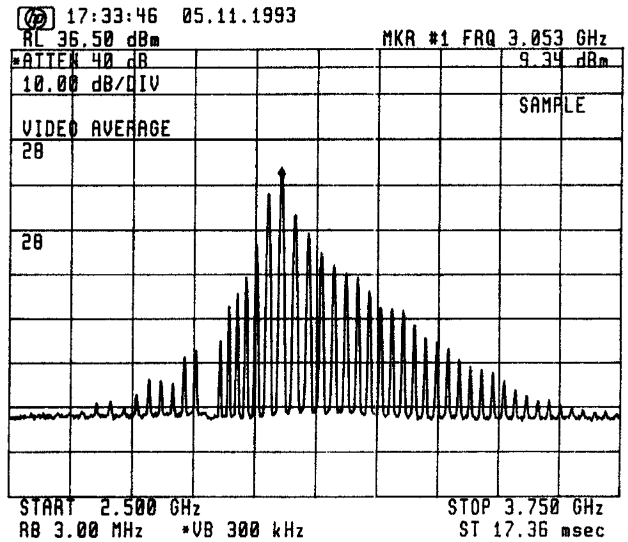
A 6- to 3-GHz monolithic frequency divider [5] of the regenerative type has been simulated in its quasi-periodic operation range using the above proposed techniques.

First of all, the circuit stability borders have been traced in the two-parameter plane $\omega_{in} - P_{in}$ from a periodic analysis (ω_p=0). The resulting loci (Fig. 1) correspond, respectively, to the solutions of the system (2) for $\Omega = \omega_{in}/2$ (I-type bifurcation locus) and for $\Omega = \alpha\omega_{in}$, $\alpha \in \mathbb{R}$ (Hopf-type bifurcation locus), and they divide the plane $\omega_{in} - P_{in}$ into three operation regions [5]: periodic regime of frequency ω_{in} (region I), frequency divider (region II), and quasi-periodic regime (region III).

The circuit is now analyzed for values $\omega_{in} - P_{in}$ inside the region III for which ω_p corresponds to the autonomous



(a)



(b)

Fig. 3. Output spectrum for input frequency 6 GHz and input power -2 dBm. (a) Simulation. (b) Measurement.

frequency, obtaining from the solutions of (2) with Ω=0 another stability border (QP locus). This is a turning-point curve that indicates the existence, for input frequencies up to 5.8 GHz, of a an hysteresis phenomenon when the transformation quasi-periodic to periodic takes place. This fact is confirmed by the experimental points drawn in Fig. 1, which show a good agreement with the simulation, except for a frequency shift due to the inaccuracy of the transistor model.

The above conclusions are evidenced in the output power characteristic of Fig. 2, obtained as a function of the input power for a constant input frequency value f_{in}=4.75 GHz. For the quasi-periodic response (self-oscillating mixer) two different paths have been traced, corresponding, respectively, to the output power variation at the external and autonomous frequencies. The autonomous frequency is of course modified along the path. The straight line corresponds to the periodic simulation of fundamental frequency ω_{in}. This curve is unstable until a Hopf-type bifurcation H appears [4], giving place to a quasi-periodic solution. The periodic path is thus stable from the point H onwards. A turning point T divides the quasi-periodic path into two different regions from the stability viewpoint. The sections HT and HT' are unstable, the rest of the branches being stable. Physically, the turning point (T and T') in the quasi-periodic branch causes a hysteresis of the circuit response. That is, when starting from a stable operating point of the periodic path and reducing the input generator value, the apparition of an autonomous frequency at point H (Hopf-type bifurcation) causes a vertical jump to the quasi-periodic branch. On the other hand, when starting from a stable solution point in the quasi-periodic branch and increasing the generator value continuously, a vertical jump will take place at the turning points T and T' to the stable periodic path. These conclusions have been experimentally verified as shown in the figure.

For f_{in}=4.75 GHz, no frequency division takes place, since the path obtained by modifying the input power does not enter the I-type bifurcation locus. However, the frequency division does take place in the band 5.20–6.75 GHz for different input levels. For the input frequency f_{in}=6 GHz, the spectrum near the locking value obtained both by simulation and experimentation is shown in Fig. 3 and evidences Adler conclusions [6], i.e., the triangular shape of the spectrum and progressive rise of the line due to the external generator as the input power increases.

IV. Conclusions

A general simulation method for regenerative circuits outside of their locking ranges has been presented, allowing a deep insight into their quasi-periodic operation regime—of invaluable interest for phase-locked loops applications. The circuit behavior along the whole variation range of any of its parameters can be predicted by obtaining its solution paths both for periodic and quasi-periodic operation. The global stability portrait of the circuit is determined by tracing the different bifurcations loci on a two-parameter space, presenting here for the first time the bifurcation loci resulting from quasi-periodic simulations. The subsynchronized operation of a M.M.I.C frequency divided by two has been thus analyzed, obtaining an excellent agreement with experimental results.

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